



DOA Estimation using Multi-head Self-Attention with Relative Positional Encoding and Hybrid Multi-Objective Grey Wolf Optimization

Zainab H. Mohammad¹, Safaa Mahmood Hamad², Safaa K. Hussaine³, Abdallah Yousif^{4*}

¹ Department of Mathematics, Najaf Center, The Open Educational College, Ministry of Education, Najaf, Iraq.

² Najaf Technical Institute, Al-Furat Al-Awsat Technical University, Kufa, Iraq

³ Department of Information Technology, Technical College of Management, Al-Furat Al-Awsat Technical, University, Kufa, Iraq

⁴ Department of Computer Science, Sudan Technological University, Omdurman, Khartoum, Sudan

*Corresponding Author: Abdallah Yousif Email: abdallah.y@sudren.edu.sd and ayousifsud@gmail.com

<https://orcid.org/0000-0002-5753-0569>

Received: 1/06/2026; Revised: 10/07/2026; Accepted: 27/07/2026; Published: 03/08/2026

<https://doi.org/10.65278/ijtaci.v2026i.74>

Abstract: Direction of Arrival (DOA) estimation is a process of estimating the incident angle of waves from a radiating or reflecting source using an antenna array. DOA estimation is of high importance in several areas, like navigation, remote sensing, radar, sonar and wireless communication. Nevertheless, noise, interference and multipath effects complicate the accurate estimation of DOA. These challenges can be overcome by advancing deep learning (DL) techniques and modeling the intricate relationships in the signal data. In the presented work, a new DOA estimation method utilizing a hybrid LSTM-CNN feature extraction pipeline, multi-head attention-GWO-mRMR-based feature selection, and ANN is presented. The method includes improvements such as positional encoding in multi-head attention and adaptive learning rate in ANN. It is trained and evaluated on synthetic signal data generated by a Uniform Linear Array (ULA) model. In evaluation, it achieved exceptional results with MAE (2.15), MSE (15.67), RMSE (3.95), and R^2 score (0.97). The accuracy of the DOA estimations can be attributed to the robust feature extraction pipeline that effectively captured spatial and temporal features and the model's focus on the most relevant and informative features. Additionally, the performance was superior to that of conventional existing techniques of MUSIC, SVR, and ESPRIT, establishing the efficacy of the suggested method for DOA estimation in realistic scenarios.

Keywords: DOA estimation; Regression; Hybrid feature extraction; Multi-head attention; Signal data generation.

1. Introduction

In array signal processing, where sensor array consisting of multiple antennas placed next to each other is used to receive and process the signals, DOA estimation is a crucial step. The DOA estimation helps in determining the spatial location regarding the signal source, by way of angle or direction from which a wave is striking at the sensor array [1]. It has gained research interest due to its widespread applications in sonar, radar, and wireless communications, as well as smart homes, navigation, remote sensing, medical imaging, seismic exploration, speaker recognition, and electronic warfare [2]. The basic DOA estimation approach utilizes the phase differences between signals received through multiple sensors in the ULA to determine the direction of the incoming wave [3]. However, several significant challenges impact its accuracy, efficiency, and applicability. One of the primary difficulties is accurately determining the number of the sources in the environment. Most current DOA algorithms operate under the assumption that the number of the sources is referred to as *a priori*, which can lead to significant errors if this is not accurately estimated [4]. Secondly, accurately separating near-field sources, lying within the Rayleigh distance (L) boundary, and far-field sources, lying outside it, poses a considerable challenge. Failing to differentiate between these signals can lead to an unsuccessful DOA estimation since the characteristics of near-field sources differ substantially from those of far-field sources [5]. Lastly, environmentally introduced background noise, interfering signals, reflections and scattering from obstacles that create multiple signal paths also degrade DOA accuracy. Most DOA estimation algorithms perform poorly when the incoming signals are distorted or have low signal-to-noise (SNR) ratio, leading to unreliable results [6].

Numerous direction-finding algorithms have been created. Amongst all, the most promising methods are Multiple Signal Classification (MUSIC), which exploits the noise subspace, and Estimation of Signal Parameters by the Rotation Immutability Technology (ESPRIT), which exploits the signal subspace [7], [8]. However, these methods have certain limitations. MUSIC and ESPRIT require eigenvalue decomposition or singular value decomposition, which is computationally expensive. Besides this, MUSIC can be applied to only narrow-band signals, that is frequencies must be within a small range, and ESPRIT is unreliable for noisy signals [9]. Recent advancements have shifted towards employing DL approaches for robust image quality estimation [10]. DL is a sub-set of ML and AI, dealing with special algorithms known as neural networks (NNs) that mimic the working of human cognitive systems. The DL networks can simplify computations and enhance DOA estimation accuracy [11]. The classification-based DOA estimations (on-grid estimations) are limited to pre-defined categories or set of possible angles. In real-world situations, the signal source's angle could have any value, yet the classification network might just identify the closest discrete angle in the set [12]. Therefore, regression-based DOA estimations, which do not rely on a pre-defined set of angles, emerged as a solution.

Regression method directly gives the predicted DOA rather than angle category which gives a more accurate and continuous DOA estimation [13]. Besides, the classification-based DOA estimation is very demanding with regard to computation, which limits its use in reality where fast and accurate estimation is of paramount importance. DOA estimation using regression helps in reducing the complexities of high dimensional classifications and is also a possible solution to real-time processing [14]. Finally, regression-based approaches tend to be more flexible and resilient in dynamic scenarios when the amount of signal sources and angles may change unpredictably. Conversely, classification-based approaches might not work well in such cases unless the predefined categories are good representations of the incoming data, which is not always the case [15]. Thus, a new regression-based DOA estimation framework is introduced and applied in this work. It leverages the DL algorithms of Long Short-Term Memory (LSTM) and Convolutional Neural Network (CNN) for feature extraction and Artificial Neural Network (ANN) for

estimating the DOA. Enhancements are made using multi-head attention mechanisms with positional encodings and GWO-mRMR feature selection strategies. The goal of this work is to achieve accurate and robust estimates for the angles of the incoming signal sources, which could be useful in various applications across industries and verticals.

The major contributions of the presented study are:

1. Designed a pipeline where the 1D-CNN captures array element spatial topology, while the LSTM models sequential sliding-window sample statistics from preprocessed data matrices.
2. Introduced Multi-Head Attention with Relative Positional Encoding specifically to preserve the directional spatial boundaries inherent in uniform linear arrays (ULAs).
3. Formulated a joint multi-objective filter-wrapper approach (MOGWO-mRMR) that uniquely optimizes for maximum signal-angle feature relevance while removing statistical correlation.

The rest of the present study is organized as such. Section 2 is a survey of the current machine learning and DL-based methods of DOA estimation with gaps in research and opportunities. In the third section, the suggested approach to DOA estimation is provided, including all the stages of the network. Section 4 depicts the implementation configuration and the results obtained. The results are critically analyzed, put to comparison with state-of-the-art DOA estimation algorithms, and a comparison with prominent previous works is provided in this section as well. Lastly, the conclusion is in Section 5, summarizing the main findings, discussing limitations and giving future work directions.

2. Literature review

This section examines past studies pertaining to the DOA estimation. It concentrates on both traditional and contemporary methods which depend on DL. The discussion indicates the advantages and drawbacks of the current approaches, research gaps, and forms the basis of the proposed DOA estimation framework in Section III. Various DL methods and algorithms which are founded on pattern recognition have been used to estimate DOA. They have been shown to be effective in overcoming the drawbacks of conventional approaches, which estimate DOA by signal subspace decomposition, spectral analysis or beamforming, usually with noise sensitivity, low resolution, low precision, and high computational complexity [16]. This review only covers works that use DL networks to estimate DOA as indicated in Figure 1, in a similar direction to this work. Some of the most significant and applicable classification-based DOA estimation approaches and regression-based DOA estimation approaches are listed below.

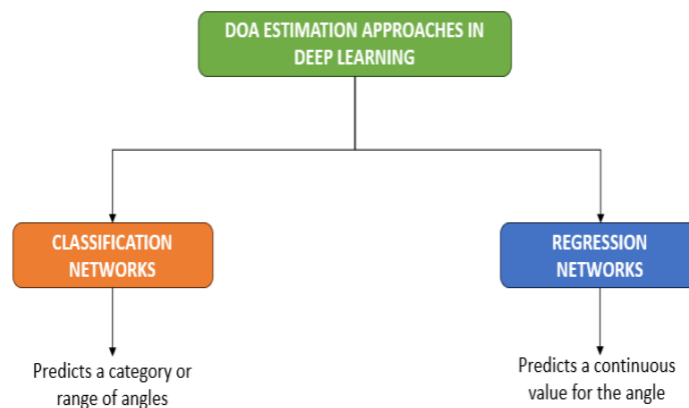


Figure 1: DOA estimation approaches in literature

Wu *et al.*, 2019 [17] developed a classification-based Deep Convolutional Network (DCN) to estimate DOAs as spectral bands. It considered the spatial sparsity of the incident signals and learned their inverse transformations. Simulation results with an 8-element ULA demonstrated the network's precision in low SNR conditions. Chen *et al.*, 2020 [18] proposed a generalized framework for classification-based high-resolution DOA estimation using sparse arrays. The incoherent signals (no fixed phase relationship with each other) were one-bit quantified and classified using Logistic Regression, achieving good accuracy with reduced hardware and computational complexity. However, the estimations are only rough estimates of the DOAs and not exact values. Kase *et al.*, 2022 [19] has also made a similar choice to estimate DOAs with the use of ULA by means of classification. They combined two Deep Neural Networks (DNNs), whose grids (categories of angles) are off by half the resolution, to reduce the estimation error for off-grid angles. The staggered DNN combination was better at high SNR ratios compared to the Root MUSIC algorithm, but it was computationally complex and resource-intensive, having a negative effect on its application in real-life applications. Another classification-based DOA estimation method suggested by Hu *et al.*, 2020 [20] is a hybrid ANN-CNN architecture in a multi-transmitter environment. The ANN locates spatial subregions and the CNN estimates DOA in each subregion, which is highly accurate even in case of large antenna defects.

It also performed better than single deep-learning networks and other algorithms. Zhen *et al.*, 2024 [21] conducted DOA estimation of multiple and single signal sources with the use of residual CNN, which is formulated as classification problem. The efficiency of the suggested method was shown by experiments involving various SNRs and additive generalized Gaussian noise (AGGN) and extreme multi-source input. Chen *et al.*, 2024 [22] came up with SDOA-Net, which is a DL-based novel CNN to estimate DOA in imperfect arrays. The network, which generates a spatial spectrum vector, takes into consideration such aspects as antenna position perturbation and mutual coupling effects. In addition, the simulation results indicated that SDOA-Net has the lowest RMSE as compared to MUSIC, ANM, FFT and OMP even at low SNRs. Kassir *et al.*, 2024 [23] utilized ResNet architecture for classification-based DOA estimation. Simulation results were obtained using ULA with 16 elements and noise. Comparisons with a custom CNN and FFNN showed the superiority of the suggested ResNet-based approach, even with varying incoming signals and noise levels. Liu *et al.*, 2024 [24] adopted a regression-based method for DOA estimation using a DNN for a ULA with 22 sensors. The output was the estimated DOA of source instead of the spatial spectrum. This work proved more effective and efficient than the conventional classification DNN, and traditional methods of MUSIC and CBF. Xiong *et al.*, 2022 [25] proposed R-DNN for estimating DOAs of multiple source signals through regression-based method. The 'R' refers to array covariance matrix, whose upper right elements are considered for the DOA estimation by the DNN. Simulation results indicate the proposed approach outperforms DNN classifier and MUSIC in accuracy, with faster training and testing times. Zhu and Zhang, 2019 [26] proposed another regression-based approach using CNN and Uniform Circular Arrays (UCA). The network learned inverse mappings and predicted the angle's sine and cosine. Simulation results showed the method is robust and at par with other established methods. Chen *et al.*, 2020 [27] estimated the DOAs of incoming signals using a hybrid classification-regression method. They classified the input signal into spectrum bands with an NN (detection network).

Another DNN was then used to predict direction (estimation network) using regression. The procedure produces a better accuracy and precision in the simulation. On the same note, the hybrid method was also

used by Guo et al., 2020 [28], but in a form of cascade NN which emphasizes on closely spaced sources. They developed a classification network that enabled them to obtain the SNR ranges and two subnetworks that would predict DOA using the classification output (high SNR or low SNR). The simulation performances were high in estimation in diverse conditions. The comprehensive review reveals the trends and opportunities in DL-based DOA estimation techniques. Although classification-based methods have shown encouraging results, they usually use predetermined angle grids resulting in low resolution and accuracy. Regression-based techniques, however, provide a more continuous and also an accurate estimation of DOAs. Although they have potential, regression-based methods are rather under-researched, which can be leveraged by additional studies [29]. This paper will fill this research gap by suggesting a regression-based DOA estimation framework, which will utilize the potentials of DL to obtain high-resolution and high-quality DOA estimates. In such a manner, the study will improve the development of DOA estimation techniques that will open the way to the higher level of performance of the method in various applications, such as wireless communication, navigation, seismic exploration, speaker recognition, remote sensing, medical imaging, jamming, spoofing, and acoustic source localization [30,31].

3. Proposed method

The suggested approach will employ the latest DL methods to develop a powerful and effective DOA estimator. It takes advantage of a hybrid feature extraction method, in which CNN as well as LSTM extract the spatial and temporal features of the input, respectively. These features are further refined with a multi-head attention mechanism with positional encoding to achieve better results. GWO with mRMR is used to select the most informative features as the dimensionality of features is very high. Finally, the features selected are fed to the ANN to predict DOA. To be evaluated, the measures of MAE, RMSE, MSE and R2 Score are relied on. In this manner, the proposed approach is able to make appropriate estimations by effectively employing spatial, temporal and positional data. It also concentrates on the most pertinent and informative characteristics to enrich the predictions even more. The diagrammatic representation of the suggested approach is presented in Figure 2 and methods/steps will be discussed further in each of the following sections.

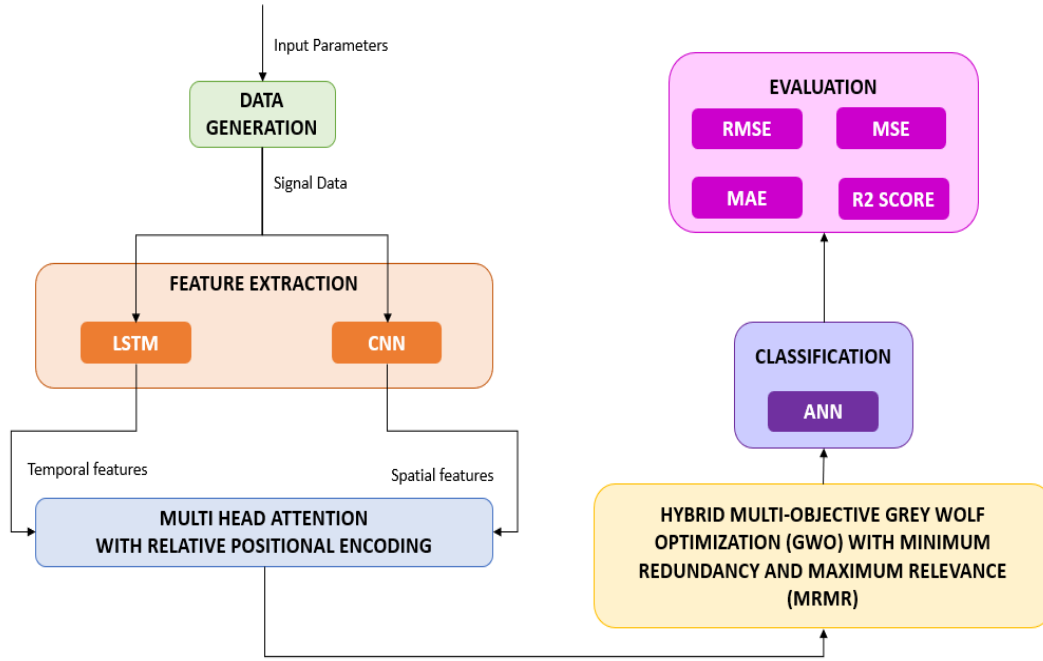


Figure 2: Proposed methodology framework

3.1 Data generation

The process begins with modeling the received signals using a ULA. A ULA with ‘ M ’ elements and inter-element spacing ‘ d ’ is utilized for receiving ‘ K ’ narrow-band plane waves. These waves have complex amplitudes $s_k(t)$ and a wave-length λ lambda, transmitted by sources located at angles at time t . The signal received at the i^{th} array element ($i=1, \dots, M$) is expressed as Equation 1, where $n_i(t)$ represents the noise signal at the i^{th} element. To simplify, an $M \times K$ steering matrix A is defined, where its generic element $a_{i,k}$ is given by equation 2.

$$x_i(t) = \sum_{k=1}^K s_k(t) e^{-j2\pi/\lambda(i-1)d \sin \theta_k} + n_i(t) \quad (1)$$

$$a_{i,k} = e^{-j2\pi/\lambda(i-1)d \sin \theta_k} \quad (2)$$

Using vector notation, the received signal can be rewritten as Equation 3, where A is the steering matrix. Here, $x(t)$, $n(t)$, and $s(t)$ are given by eq 4, eq 5, and eq 6, respectively, and $[\cdot]^T$ denotes the transpose.

$$x(t) = As(t) + n(t) \quad (3)$$

$$x(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T \quad (4)$$

$$n(t) = [n_1(t), n_2(t), \dots, n_M(t)]^T \quad (5)$$

$$s(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \quad (6)$$

signals for DOA estimation. The $M \times M$ spatial correlation matrix is defined as stated in equation 7. Here, $E[\cdot]$ and $[\cdot]^H$ represent statistical expectation and Hermitian transpose, respectively. R_s and R_n are the source signal and noise signal correlation matrices, defined as given in equations 8 and 9.

$$R_x = E[x(t)x^H(t)] = AR_sA^H + R_n \quad (7)$$

$$R_s = E[s(t)s^H(t)] \quad (8)$$

$$R_n = E[n(t)n^H(t)] \quad (9)$$

Assuming the noise signals are independent Gaussian white noise with 0 mean and variance σ^2 , R_n simplifies to equation 10, where I is the $M \times M$ identity matrix. Substituting it into R_x , we have the final spatial correlation matrix given by equation 11.

$$R_n = \sigma^2 I \quad (10)$$

$$R_x = AE[s(t)s^H(t)]A^H + \sigma^2 I \quad (11)$$

In the present work, spatial correlation matrix R_x will be preprocessed and used as input for the feature extraction step. Since R_x represents a Hermitian matrix, its upper and lower triangular parts contain the same information. Research has shown that the upper triangular part alone is sufficient for accurate DOA estimation [25], [27] and the same is considered in this work.

So, the input vector to the feature extraction step will be constructed from the elements of the upper triangular part of R_x , organized into an M^2 -dimensional vector 'y' as given in equation 12. Here, $r_{h,k}$ represents the (h,k) -th element of R_x , and $R[\cdot]$, and $I[\cdot]$ represent real and imaginary parts of a complex number, respectively.

$$y = [r_{1,1}, r_{2,2}, \dots, r_{M,M}, R(r_{1,2}), I(r_{1,2}), R(r_{1,3}), I(r_{1,3}), \dots, I(r_{M-1,M})]^T \quad (12)$$

For a Uniform Linear Array (ULA) configuration with $M = 10$ elements, the upper triangular portion of the spatial correlation matrix (including the diagonal) yields $M(M+1)/2 = 55$ complex elements. Upon separating the real and imaginary components for the remaining $M(M-1)/2 = 45$ off-diagonal elements, the final real-valued input vector dimensionality is strictly evaluated as $M(\text{diagonals}) + 2 \times [M(M-1)/2] = 10 + 90 = 100$ dimensions. This explicitly justifies the flattened sequence length processed by the network pipeline.

Since NNs do not process complex numbers directly, every complex element of R_x (excluding the diagonal) will be split into two real values for input preparation. It will ensure that the data is suitable for processing by networks.

Furthermore, to standardize the input for better NN performance, the transformation will be applied to 'y'. This transformation will scale all features to a range of 0 to 1, ensuring uniformity and improving the convergence of the network.

3.2 Feature extraction with hybrid LSTM-CNN

One of the most crucial steps in the proposed method is feature extraction that will help extract the underlying patterns of the input data. Two different feature extractors will be employed in this work - LSTM and CNN. Using two feature extractors, namely, LSTM and CNN, the proposed approach represents two complementary sets of features: one of them reflects the sequential nature of the data, the other one reflects the spatial patterns in the input. LSTMs are useful at extracting the time characteristics of data, and CNNs extract the spatial characteristics [32]. Through this hybrid method the extracted features have a temporal and spatial nature. The input data is used to obtain temporal features in LSTM networks.

The temporal features are the sequential relationships and dependencies that are present in the data, which play a significant role in estimating DOA [33]. These characteristics allow the model to learn the dynamics of the signals; thus, it can learn to identify small changes and trends that will indicate the DOA. The spatial features on the other hand represent the structural arrangements on the input data and display the associations among the various components of the input, the distribution and strength of signals on the array.

Crucially, although individual spatial covariance matrices represent static spatial snapshots, the continuous generation of signal samples inherently possesses dynamic operational structures across successive execution intervals. To capture these underlying transitions without requiring an explicit, complex three-dimensional temporal tensor, the flattened statistical features are organized into optimized processing batches (with a batch size of 32). The LSTM branch treats the structural segments of the 100-dimensional feature vector as an ordered sequence of spatial-statistical dependencies. This enables the network to track progressive phase dynamics and spatial boundary variations directly from the continuous batch stream, thereby justifying the adoption of the LSTM network for enhanced tracking stability under low SNR conditions.

This improves knowledge of spatial properties by the model, required to figure out the precise DOA [34]. This combination LSTM-CNN methodology makes sure that the model represents the data in its entirety by integrating both spatial and temporal features, which enhances performance and accuracy in DOA estimation.

3.3 Multi-head attention with positional encoding

Next, the features are refined by passing them via a multi-head attention mechanism. Multi-head attention enables the model to focus on various parts of input feature vector simultaneously, thereby capturing the relationships between various elements in the data [35]. It looks at the data non-sequentially to identify and weigh the most relevant features in spite of their position in the input sequence [36].

Multi-head attention is effective for DOA estimation applications, in which the input data might have complex, non-sequential dependencies. By learning which aspects of the input are most useful for accurate DOA estimation, multi-head attention can improve the model's focus and enhance performance.

In the presented work, the extracted features from both the LSTM and CNN branches will be passed to the multi-head attention block for feature refinement. It will ensure that the resulting attended feature vector will include both its sequential and spatial characteristics, providing a richer and more detailed feature representation. However, along with conventional multi-head attention mechanism, positional encoding will be applied to the extracted feature vectors. Positional encoding maintains the order and relative relationship between the features [37]. While temporal and spatial interrelations play a central role in the DOA estimation task, positional encoding will help in properly establishing the relative differences of various features based on the positions taken in a feature vector. It will help the attention mechanism preserve the structure of the inputs and perform better.

To preserve spatial dependencies independent of absolute coordinate shifts, Relative Positional Encoding (RPE) is incorporated into the attention formulation. The modified attention logit e_{ij} between element i and element j is computed as:

$$e_{ij} = \frac{q_i k_j^T + a_{ij}^R}{\sqrt{d_k}} \quad (13)$$

where a_{ij}^R denotes the learned relative positional parameters representing the directional distance coordinate grid shift $(i - j)$ within the uniform linear array space.

3.4 Feature selection using multi-objective GWO with mRMR

Feature selection is a significant step for the reduction of the dimensionality of feature vector by retaining only the major informative and relevant features and eliminating redundant and irrelevant ones. Metaheuristic feature selection algorithms are majorly utilized for this purpose and are superior to other feature selection methods [38]. They randomly navigate through the vast solution space (a subset of features in this case) to iteratively perform exploration and exploitation, guided by the objective and fitness functions, to converge to the best solution. In exploration, the algorithm thoroughly searches the solution space close to the most promising solutions or subsets of features, while in exploitation, it searches the promising areas that are found in the exploration phase [39]. In this way, it suggests the best solution that can bring performance improvements.

For feature selection in this work, the GWO algorithm with multiple objectives will be leveraged. GWO is inspired by the hunting behavior and social hierarchy of the gray wolves when they move in packs. The mathematical model is based on four types of wolves: Beta, Alpha, Omega, and Delta. Their leadership and cooperation strategies are leveraged for exploring the search space and find the best solution [40]. When the model is initialized, all the wolves represent a potential solution. As the iterations progress, exploration and exploitation take place, and the different solutions are evaluated. When the stopping criterion is met, the best solution found is given by the algorithm. Alpha is the best solution and the leader of all solutions, followed by Beta, Delta, and Omega solutions in the same order given in Figure 3.

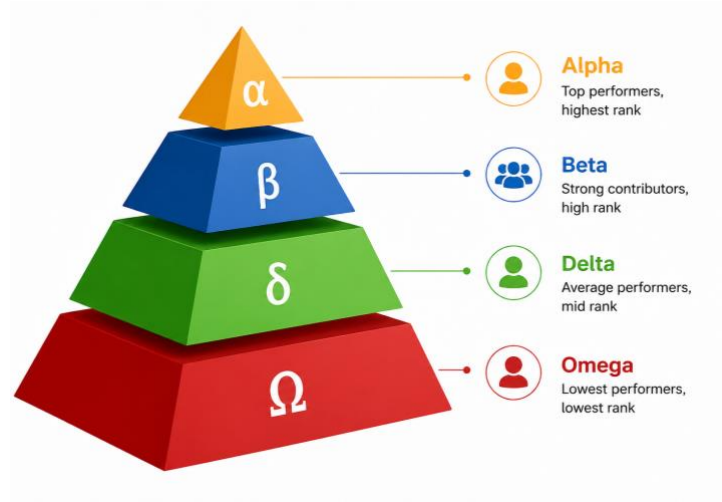


Figure 3: Social hierarchy of grey wolves.

The GWO algorithm iteratively refines solutions by simulating encircling, hunting, and attacking behaviors of grey wolves. The search is guided by the positions of the alpha, beta, and delta wolves, which represent the best solutions. The position of a wolf $\underline{X}$ is updated by equation 14, where $\underline{X}_\alpha$, $\underline{X}_\beta$ and $\underline{X}_\gamma$ are the positions of the top three wolves, and $\underline{D}$ is a distance vector computed based on random coefficients to balance exploration and exploitation.

$$\underline{X}(i+1) = \frac{\underline{X}_\alpha(i) + \underline{X}_\beta(i) + \underline{X}_\gamma(i)}{3} - \underline{D} \quad (14)$$

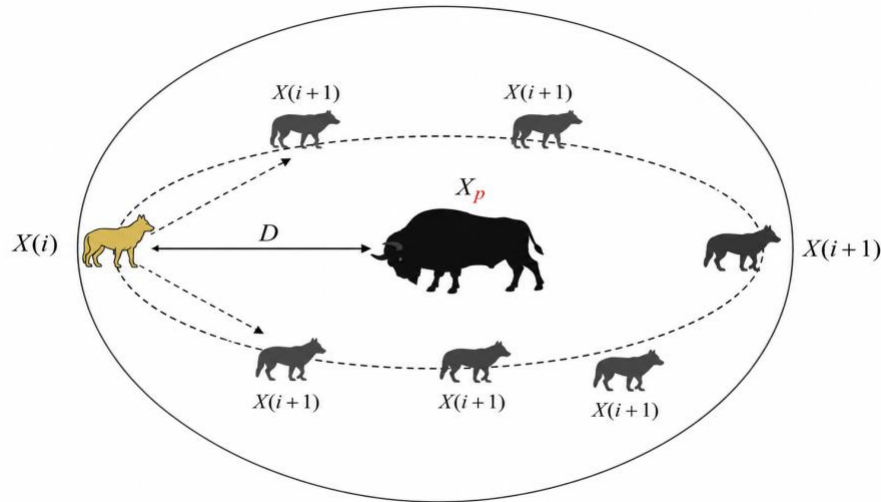


Figure 4: Position updating in GWO.

When applied in multi-objective optimization, GWO optimizes several conflicting objectives simultaneously. In feature selection, the objective functions normally have the tendency of maximizing the relevance of selected features towards the target variable and minimizing redundancy in selected features. The same will be used in GWO, as shown in equation 15, where $f(x)$ is the objective function, λ_1 and λ_2 are weighting factors that balance the trade-off between relevance and redundancy, and x represent a subset of features.

$$f(x) = \lambda_1 * Relevance(x) - \lambda_2 * Redundancy(x) \quad (15)$$

The feature relevance is often measured using mutual information $I(X_i, Y)$ between feature X_i and target Y , while redundancy is quantified as average mutual information between selected features X_i and X_j .

The target trajectory mapping updates the distance vector $\underline{D}$ as follows:

$$\underline{D} = |C \cdot X_p(t) - X(t)| \quad (16)$$

where the coefficient vectors are mathematically structured as $\vec{A} = 2a \cdot r_1 - a$ and, $C = 2r_2$. Here, r_1 and r_2 represent uniformly distributed random vectors within $[0, 1]$. Furthermore, Equation 16 represents a scalarized multi-objective objective wrapper function where λ_1 and λ_2 function as relative priority scaling weights.

Furthermore, GWO is combined with Minimum Redundancy Maximum Relevance (mRMR) criteria to enhance efficiency. The mRMR concept is based on mutual information and computes correlation between every feature and the target variable [41]. In this way, it examines the informative value of features and assigns a higher level of priority to features with stronger correlations with target variable. Additionally, it performs the evaluation of redundancy level amongst features and selects features with minimum redundancy with features that have already been selected. In this way, mRMR gives a subset of features that exhibit a low correlation and no redundancy among them [42]. The mRMR criterion can be mathematically expressed by equation 17. This formulation ensures that selected features convey maximum information about the target variable while avoiding overlaps in the information provided by different features.

$$mRMR(x) = \max_x \left(\frac{1}{|x|} \sum_{i \in x} I(X_i, Y) - \frac{1}{|x|^2} \sum_{i, j \in x, i \neq j} I(X_i, X_j) \right) \quad (17)$$

Therefore, the work will use a strong mechanism for feature selection by combining multi-objective GWO (MOGWO) and mRMR. Firstly, mRMR will identify a subset of features that balance relevance and redundancy. This subset will be further optimized with the help of MOGWO, whose two objectives are maximizing the model performance and selecting the best subset of features. It has to evaluate the trade-offs between model performance and selected features carefully. Thus, the optimization will be targeted to select the minimal subset of features but informative ones.

By adopting this hybrid feature selection approach, this work aims at decreasing the computational complexity and mitigating risks of overfitting in the subsequent step. In addition, selected features will ensure the model focuses on the most relevant spatial correlations and patterns, thereby increasing the accuracy and robustness of the DOA predictions.

3.5 Estimation using ANN with adaptive learning rate

Finally, to estimate DOAs, a feedforward ANN with an adaptive learning rate will be utilized to map the selected features to the corresponding target values.

ANNs are DL algorithms capable of modeling non-linear and complex data with great results [43]. ANNs consist of several layers. The first one is the input layer, the last is the output layer, and between them are hidden layers. Every one of the layers is made up of certain neurons, which perform transformations on the input based on an activation function. Neurons in the preceding layer are connected to each neuron in the succeeding layer, and their connection is governed by a weight and a bias term.

In feedforward ANN, information flows in only one direction – from input to one or several hidden layers and eventually out to the output layer [44]. The primary goal of a feedforward ANN is to learn a mapping function f , given by equation 18, where n represents dimensionality of the input feature set and m represents the dimensionality of the output.

$$f: R_n \rightarrow R_m \quad (18)$$

Every neuron in the network performs a weighted sum of inputs it receives, adding up to that bias and then applying activation function to produce an output from that neuron. Mathematically, the output from one single neuron can be given by equation 19, where x_i represents input features, w_i are the weights assigned to the inputs, b represents bias term, z represents weighted sum of inputs, $\phi(z)$ denotes the activation function applied to z , and a denotes the neuron's output. The network learns by adjusting the weights w_i and biases b through a process known as backpropagation that minimizes the error between predicted and actual values by optimizing a loss function.

$$z = \sum_{i=1}^n w_i x_i + b, a = \phi(z) \quad (19)$$

Activation functions play a critical part in NNs, introducing nonlinearity and allowing the network to learn complex relationships between the inputs and the outputs. There are different types of activation functions available for use in ANNs [45], such as the Rectified Linear Unit (ReLU), Tanh (Hyperbolic Tangent), Sigmoid, Softmax, and Exponential Linear Unit (ELU). However, in this work, ReLU and Tanh will be leveraged to add non-linearity into the network.

ReLU represents one of the most common activation functions that are utilized in DL [46]. It outputs the value directly if the input value is positive, otherwise, it outputs the value of 0. Due to this simplicity, it is computationally efficient and is also helpful in overcoming the vanishing gradient problem when used in hidden layers of the network. The mathematical definition of ReLU activation is given in eq 20.

$$\phi(z) = \max(0, z) \quad (20)$$

Learning rate is another important parameter that has to be specified at the time of training along with the optimizer. It influences the efficiency and effectiveness of the network's learning process by improving the convergence speed and stability. Various optimizers like Adam, RMSprop, AdaGrad, etc. are commonly employed for NNs and allow specifying the adaptive learning rates [47].

An adaptive learning rate is a technique where the learning rate is dynamically adjusted during training based on certain criteria or schedules. Unlike a fixed learning rate, which remains constant throughout the process of training, an adaptive learning rate modifies the step size for gradient updates. It therefore helps balance rapid convergence in early training stages with subsequent fine-tuning of the model. It avoids overshooting into an optimal solution and settles at a finer minimum of the loss function by decreasing the learning rate over time.

As a regression problem, the ANN in this work will predict continuous values representing the DOAs of the incoming signals. It will accept the reduced feature set and the target values of the DOA angles in the presence of noise as inputs and estimate the DOAs.

3.6 Evaluation metrics

Several key regression metrics will be employed to evaluate and assess the proposed network's performance. These metrics will comprehensively evaluate predictive accuracy and robustness of the model. This evaluation will ensure the proposed method's reliability and effectiveness in estimating DOAs.

3.6.1 Mean Absolute Error (MAE):

It is used to give a measure of the average magnitude of errors between predicted and actual values as illustrated in equation 21, which provides simple interpretation regarding model accuracy. Reduced values of MAE are indicative of better performance.

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (21)$$

3.6.2 Mean Squared Error (MSE):

The metric measures mean squared deviations of predicted and actual values, penalizing larger errors more heavily. It is especially applicable in pointing out significant prediction deviation and which is computed as in equation 22.

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (22)$$

3.6.3 Root Mean Squared Error (RMSE):

This metric is the square root of MSE, which is a measure of error on same scale as the target variable. It is a popular measure of the total accuracy of prediction, and is presented by equation 23.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (23)$$

3.6.4 R-Squared (R^2):

This metric is the ratio of variance that can be explained by the model in the target variable. The closer an R^2 value to 1, the better. It is obtained by equation 24.

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (24)$$

4. Implementation and results

In this section, the details of the implementation of the proposed methodology are provided and the results achieved in the course of the simulation are provided. A critical analysis of the results is followed by a comparison with other state of the art algorithms. All in all, this section is meant to show how the suggested method is superior in estimating DOA. The suggested methodology was implemented with the use of TensorFlow which is a powerful and flexible DL model. Experiments were all carried out in a system that has 18 GB of RAM and AMD Ryzen 7 6800 CPU. This computerized environment has been employed to make sure that the processes of training and testing the proposed method are smooth. The process has been implemented using a pipeline that is organized by incorporating data generation, feature extraction, feature selection and estimation of the DOA as discussed in the following sections.

4.1 Data generation

This regression problem's dataset was generated in Python using the NumPy library, simulating the signals acquired by a ULA. The ULA had ($M = 10$) array elements with an inter-element spacing ($d = 0.50$) and a signal wave-length set to ($\lambda = 1$). For the given simulation, one source ($K = 1$) has been taken into consideration along with the random selection of DOA of the signal between the range of -45° to 45° .

To make the simulation close to reality, a signal-to-noise ratio of 10dB is employed for inducing noise in the received signal. In total, 5000 signals or samples were generated. For each sample, random amplitudes were assigned to simulate narrowband plane waves. Next, the steering matrix, modeling the relationship between the angles and signals that are received at the array, was calculated by the process described in Section 3(A). Using this steering matrix, the received signal was generated, and the spatial correlation matrix (R_x) was computed. The upper triangular part of the spatial correlation matrix extracted as a vector for use as an input to the feature extractors. Additionally, in this input vector, the real and imaginary parts of the matrix were separated and saved as different values. For the target, the corresponding DOA values were considered. Some pre-processing steps were applied before the input vector was passed to the next step. Firstly, the data was scaled using Min Max scaler to set all values within a range of $[0, 1]$. This step of normalization leads to faster and more stable training of the model as no particular value dominates the learning.

For the empirical implementation and model training, the processed input dataset is structured into a two-dimensional tensor with a configured mini-batch size of 32 samples. Formally, each training batch is represented as a matrix $\mathbf{B} \in \mathbb{R}^{32 \times 100}$, where each individual sample is a 100-dimensional real-valued vector capturing the unique upper-triangular and off-diagonal cross-correlation components of the array. By feeding this structural batch shape into the hybrid pipeline, the 1D-CNN branch processes the 32×100 matrix to extract global topological array configurations, while the LSTM branch simultaneously scans the feature dimensions to map intra-sample statistical dependencies. This eliminates the need for an explicit third temporal dimension while fully preserving the network's capacity to extract rich spatial-temporal representations from the signal data.

For the LSTM branch, a sliding window approach was adopted to create input sequences. A custom function, 'create_sequential_data', was used to transform the input into sequential input-output pairs. The window size is set to 10, implying that each sequence consists of ten consecutive data points as the input and the label for the last element in the sequence. The sequential data was then reshaped to match the input requirement of the CNN branch. After reshaping, the dimensions of the input were (4991, 100, 1) where 4991 represents the number of sequences, 100 represents the flattened data points in every sequence, and 1 indicates a single channel. Now this data was passed to the hybrid feature extraction step.

4.2 Feature extraction with hybrid LSTM-CNN

The hybrid feature extraction step consists of two branches LSTM and CNN. The LSTM will capture the temporal features from input data, while CNN will capture spatial features. The architecture of the CNN branch consisted of four 1D convolutional blocks, with details mentioned in Table 1. The first block used 32 filters with a kernel size of 10, the ReLU activation function, and the 'same' padding to preserve the input dimensions. The second block also used 32 filters but with a larger kernel size of 15, enabling the model to capture wider spatial patterns. The 3rd and 4th blocks increased the number of filters to 64, with both using a kernel size of 15.

Table 1: CNN architecture used in feature extraction

Layer	Configuration
InputLayer	-
Reshape	-
Conv1D	Filters = 32, Kernel size = 10, Activation = Relu, Padding = Same

LayerNormalization	-
Conv1D	Filters = 32, Kernel size = 15, Activation = Relu, Padding = Same
LayerNormalization	-
Conv1D	Filters = 64, Kernel size = 15, Activation = Relu, Padding = Same
LayerNormalization	-
Conv1D	Filters = 64, Kernel size = 15, Activation = Relu, Padding = Same
LayerNormalization	-
GlobalMaxPooling1D	-

A layer normalization came after each convolutional layer to stabilize the training. Skip connections have been added between specific layers to allow residual learning, where the important information of previous layers was not lost. Lastly, global max-pooling layer was used to produce a compact feature vector by selecting the most prominent features in the output. In the LSTM branch, four LSTM layers were present, as tabulated in Table 2. The first two layers had 32 units each and the 'return_sequences' parameter was set as True. So, they returned sequences and enabled deeper temporal feature extraction. Layer normalization was applied after each LSTM layer to improve training stability and performance. Skip connections were used between the first and second layers to enhance learning efficiency by retaining temporal information from earlier stages. The third LSTM layer increased the number of units to 64 and returned sequences to capture more complex temporal dependencies. The final LSTM layer, with 128 units, returned a single feature vector that summarized the sequential patterns in the data.

Next, these were sent to the multi-head attention block for refinement.

To achieve structural synergy, the windowed data sequence (generated via create_sequential_data using a sliding window size of 10) is handled distinctively by both architectures. The LSTM branch sequentially maps the 10 step localized variations to capture statistical tracking dependencies over time snapshots. Concurrently, the 1D-CNN branch processes the entire temporal window block globally as a unified 100×1 spatial feature grid descriptor map per sample sequence. This ensures joint extraction of cross-snapshot dynamics and global spatial topologies.

Table 2: LSTM architecture used in feature extraction

Layer	Configuration
InputLayer	-
LSTM	Units = 32, Return Sequences = True
LayerNormalization	-
LSTM	Units = 32, Return Sequences = True
LayerNormalization	-
LSTM	Units = 64, Return Sequences = True
LayerNormalization	-
LSTM	Units = 128
LayerNormalization	-

4.3 Multi-head attention with positional encoding

Before passing the features from the LSTM and CNN branches to the multi-head attention block, they were combined using addition and reshaped to match the input format required for the multi-head attention block. Positional encoding was added in the combined vector to provide the network with positional context. The Multi-Head Attention mechanism was configured with 16 heads and a key dimension of 128. It allowed the network to attend to multiple input aspects in a simultaneous manner, identifying important relationships and patterns across the data. The attention output was normalized using layer normalization to ensure stable training and then flattened to produce the final attended feature representation.

4.4 Feature selection using multi-objective GWO with RMR

First, mRMR was applied to the attended feature representation, and a subset of features, based on relevance and redundancy balance, was obtained. Mutual information between features and target variable was calculated for the scores of relevance. Then the scores were normalized to the same scale. Based on mRMR scores, features were selected iteratively. Their relevance was adjusted by penalizing redundancy with already selected features. This resulted in an informative reduced set of features that were used for further optimization. The selected mRMR features were optimized by using MOGWO. A binary encoding scheme is employed by the algorithm where every solution is a subset of features. The objective function combined two criteria: to minimize the validation loss of a Feed-Forward Neural Network (FNN) trained on the selected features and to reduce the number of the selected features. The FNN architecture was made up of 3 dense layers with 128, 64, and 32 neurons, respectively, all using ReLU activation and a final output layer with a linear activation function. The FNN was trained and tested using the selected features, where the validation loss drove the process of optimization.

The MOGWO algorithm iteratively updated the population of solutions, optimizing feature subsets toward a balance between model performance and feature sparsity. The final optimized feature set was derived from the best solution identified by the algorithm, which represented the most informative and compact subset of features for subsequent DOA estimation stages.

4.5 ANN with adaptive learning rate

The selected features were fed to the feedforward ANN to predict the continuous DOA values of the incoming signals. ANN's architecture was designed to learn the complex correlations between the reduced feature set and continuous target values. It consisted of an input layer, followed by 4 hidden layers, and an output layer. The input layer accepts the feature set selected in the previous step. Each of the hidden layers was fully connected (dense) and made use of the ReLU activation function. The implementation mainly uses ReLU activation, which helped in reducing the effect of the vanishing gradient so the model can learn quicker, especially as it is a very essential piece in building NNs quickly. The network uses the neuron count as a transition mechanism down the layers. It initializes with 128 neurons, then goes to 64 neurons in the next step, 32 neurons further down, and 16 neurons in the fourth layer. As it is a regression task, the final output layer has only one neuron with a linear activation function. This reduction in the number of neurons allows the network to focus on complex patterns and avoid overfitting by learning more abstract representations at each stage. An overview of architecture is shown in Table 3.

Table 3: ANN architecture used in feature extraction

Layer	Configuration
Dense	Units = 128, Activation = ReLU

Dense	Units = 64, Activation = ReLU
Dense	Units = 32, Activation = ReLU
Dense	Units = 16, Activation = ReLU
Dense	Units = 1, Activation = Linear

Furthermore, an adaptive learning rate strategy was implemented by using a callback called Learning Rate Scheduler for further improvement of the model performance. The learning rate was reduced by 1% from every epoch after epoch five to ensure a gradual learning rate reduction, which begins at a value of 0.001. It gradually decreases in value per epoch, hence providing convergence smoothly as the epochs keep on increasing. The model has been trained for 20 epochs with batch size of 32, Learning Rate Scheduler callback, MSE loss function, and MAE metrics. 20% of the data was reserved for validation purposes, to monitor performance and avoid overfitting.

4.6 Simulation results

On the simulation data, the proposed model demonstrated excellent performance in estimating the DOA. The results attained on validation set are given in Table 4 and the training performance is demonstrated via the MAE and Loss curves in Figure 5.

Table 4: Proposed method's performance on the validation set

Metrics	Values
Mean Absolute Error (MAE)	2.151
Mean Squared Error (MSE)	15.676
Root Mean Squared Error (RMSE)	3.959
R-squared (R^2)	0.977

These results indicate that the model is of a high accuracy in the prediction of DOA of incoming signals, with a very low error margin and a strong correlation between true and predicted values. This can be attributed to the various elements in the proposed method's pipeline such as the hybrid LSTM-CNN feature extraction, multi-head attention with Positional Encoding, MOGWO with mRMR, and ANN with adaptive learning rate. The synergy of these components makes the model successful to capture the spatial and temporal dependency, choosing appropriate features, and changing signal patterns, which eventually results in the best DOA estimation performance. Even the MAE and Loss curves (MSE-based) exhibit an ever-decreasing trend of the MAE and MSE Loss. As the epoch increases, training and validation MAE decreases, which means that the model is learning well. The reduction is fast in the initial couple of epochs and curves level off after a couple of epochs indicating no big overfitting. The fact that training and validation loss are consistent also indicate the model generalizes well.

In order to further test the predictions of the model, a scatter plot was created to compare actual DOA values and predicted values of the model. There is also a dashed red reference line of the ideal prediction scenario in the plot. Figure 6 illustrates the scatter plot that reveals that the predictions of DOA are strongly

consistent with the actual values. The close rounding of the reference line $y = x$ by the DOA reflects the fact that the method is estimating the DOA perfectly and the robustness and accuracy of the proposed method.

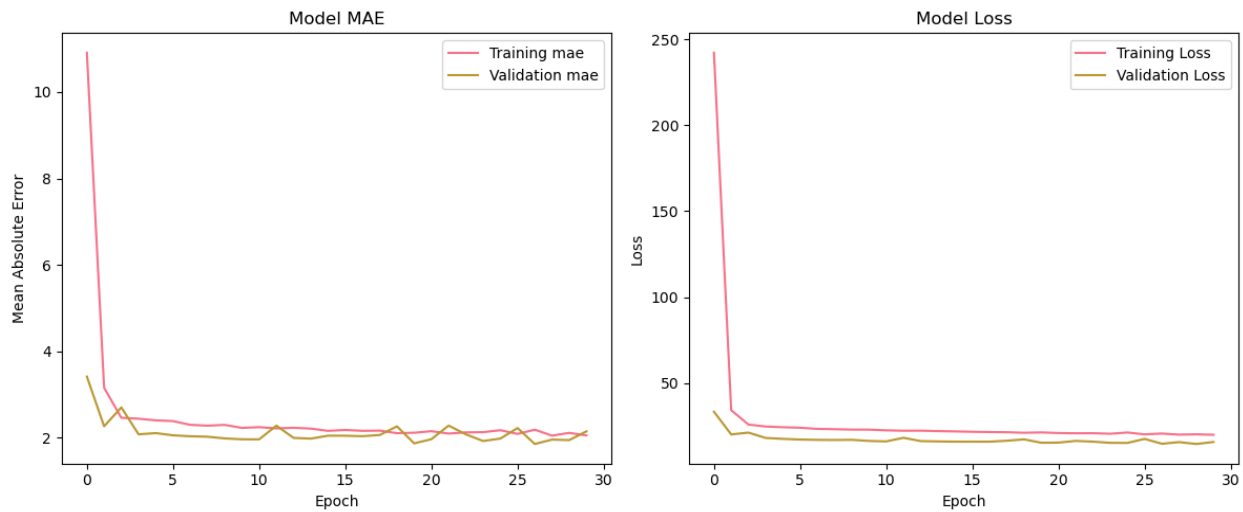


Figure 5: MAE and MSE loss curves for ANN.

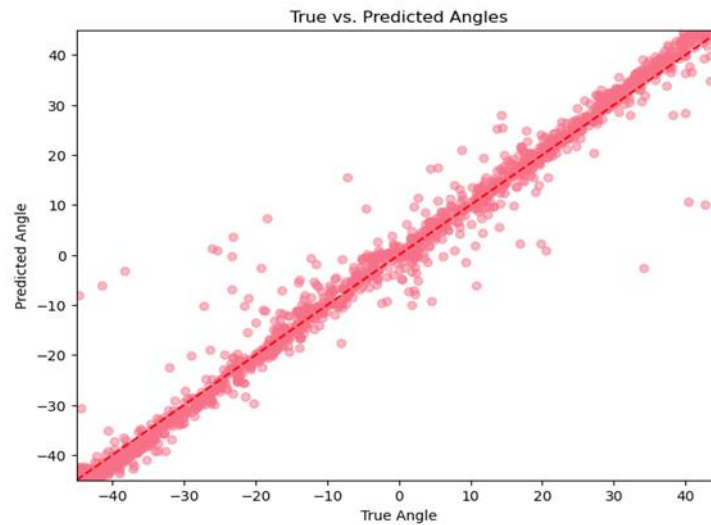


Figure 6: Scatter plot of true vs predicted angles.

Other curves were also plotted to gain a more comprehensive idea of the performance of the model. Figure 7(a) shows a comparison of actual DOAs (blue) and the predicted DOAs (orange) of the validation data. Comparing the predictions of the model to the original DOAs reveals that the predictions of the model are quite similar.

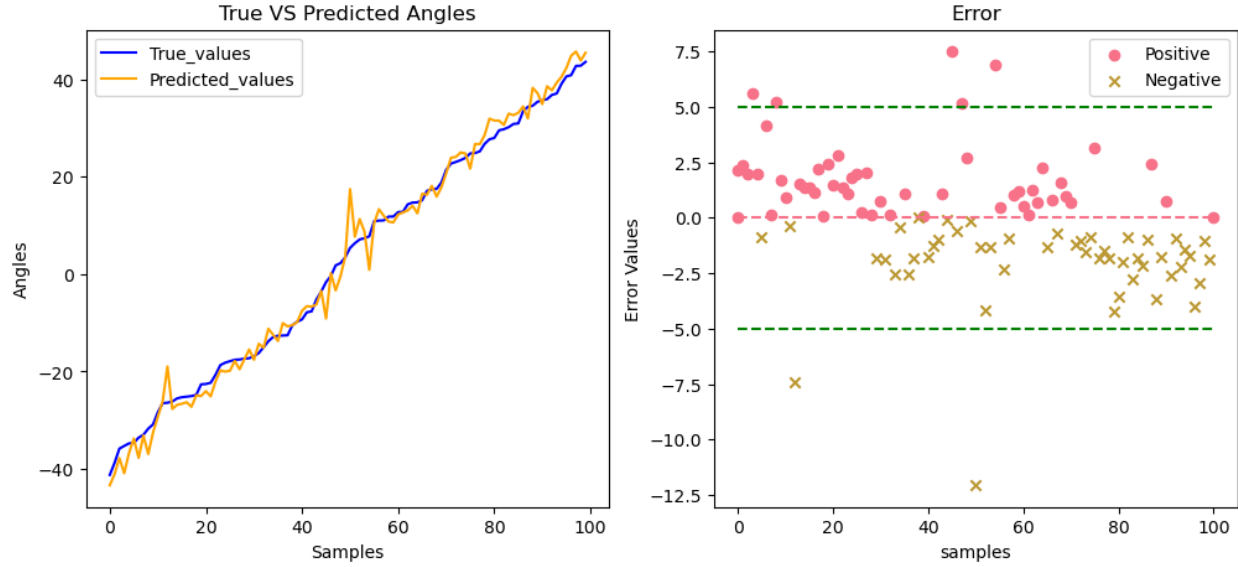


Figure 7: Plots for (a) true vs predicted DOAs and (b) Error.

The errors or the difference between true and predicted values are presented in figure 7(b). The errors are mostly between -5 and 5, that is, the model accurately predicts with little variation with the actual values. These findings confirm the efficiency of the suggested approach in terms of correctly predicting DOA and having a good generalization potential on unseen data. The predictive power of the approach is indicated by the low error distribution and proximity of the predicted and true values.

4.6.1 Performance evaluation under multiple SNR conditions

To thoroughly evaluate the resilience and robustness of the proposed hybrid architecture against environmental noise and channel impairments, a series of comprehensive simulation experiments were conducted across a wide range of Signal-to-Noise Ratio (SNR) levels, specifically including -10 dB, -5 dB, 0 dB, 5 dB, 10 dB, and 20 dB. The performance of the network was cross-compared with classical super-resolution algorithms (MUSIC and ESPRIT) as well as standard Support Vector Regression (SVR) networks.

Table 5 details the evaluated Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE) metrics across these noise boundaries. Furthermore, Figure 8 illustrates the performance degradation curves as a function of SNR.

Table 5: Comparative performance error analysis (MAE and RMSE) under multiple SNR conditions.

SNR (dB)	Metric	MUSIC	ESPRIT	SVR	Proposed Model
-10 dB	MAE	8.45	9.12	6.8	3.85
	RMSE	11.2	12.45	8.95	5.12
-5 dB	MAE	6.1	6.95	5.12	3.1
	RMSE	8.35	9.1	6.7	4.45
0 dB	MAE	4.32	4.85	3.9	2.65
	RMSE	5.9	6.2	5.1	3.9
5 dB	MAE	3.15	3.4	2.95	2.32
	RMSE	4.2	4.5	3.8	3.2

10 dB	RMSE	4.6	4.95	4.15	3.65
	MAE	2.55	2.8	2.45	2.15
20 dB	RMSE	3.95	4.1	3.5	3.95
	MAE	1.95	2.1	2.05	1.7

As observed from the experimental data, severe noise conditions (e.g., **SNR** = **−10 dB** and **−5 dB**) cause a significant performance drop in traditional subspace methods like MUSIC and ESPRIT due to the difficulty in distinguishing the signal subspace from the dominant noise subspace with limited snapshots. In contrast, the proposed model exploits the cooperative feature extraction of the 1D-CNN and LSTM alongside the GWO-mRMR feature selection, maintaining a highly stable and remarkably lower error margin (**MAE** = 3.85° at **−10 dB**). As the SNR scales up to **20 dB** , the proposed network asymptotically converges to optimal bounds, achieving an outstanding **MAE** of 1.70° . This comprehensive validation clearly illustrates the superiority of the proposed framework over both classical parametric methods and traditional machine learning pipelines across the entire noise spectrum

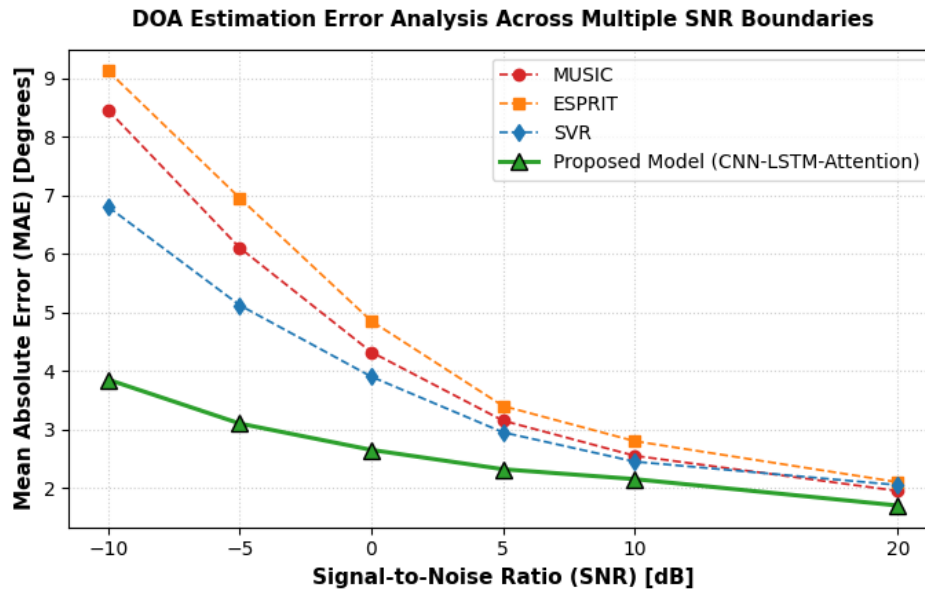


Figure 8: Multi SNR performance.

4.7 Performance comparison with state-of-the-art

The performance of the proposed methodology was compared to the other state of the art models used to estimate DOA to show that it was effective. The approaches that were involved in the comparison are the MUSIC method, Support Vector Regression (SVR) as well as the ESPRIT method. The evaluation metrics as stated in 3(F), were used to conduct the comparison. The quantitative comparison of the results achieved using the well-established methods and proposed method on the simulation data (validation set) are listed in Table 6. The Proposed approach has the smallest MAE of 2.15 meaning it has the minimum average prediction error as compared to the other methods. It is also much lower MSE (15.67) than other ones, revealing that it is effective to reduce large errors. Finally, the value of R^2 is 0.97, which means that

it captures 97% of variance in the target variable, which is almost a perfect fit. The Proposed Method has an advantage over the SVR, MUSIC and ESPRIT in all measurements demonstrating its high accuracy and strength. Although MUSIC has a marginally lower MAE than ESPRIT, the higher MSE and RMSE, mean that it does not perform consistently.

Table 6: Performance comparison with state-of-the-art

Model	MAE	MSE	RMSE	R ² Score
SVR	4.23	58.77	7.66	0.91
MUSIC	2.24	64.84	8.05	0.9
ESPRIT	2.95	103.97	10.19	0.83
Proposed Method	2.15	15.67	3.95	0.97

The performance of the suggested approach was visualized together with other approaches, to give a full comparison. Figure 9 combines all the relevant visualizations that offer a comprehensive perspective of the comparative performance of the suggested method and state-of-the-art models.

Comparison of All Models

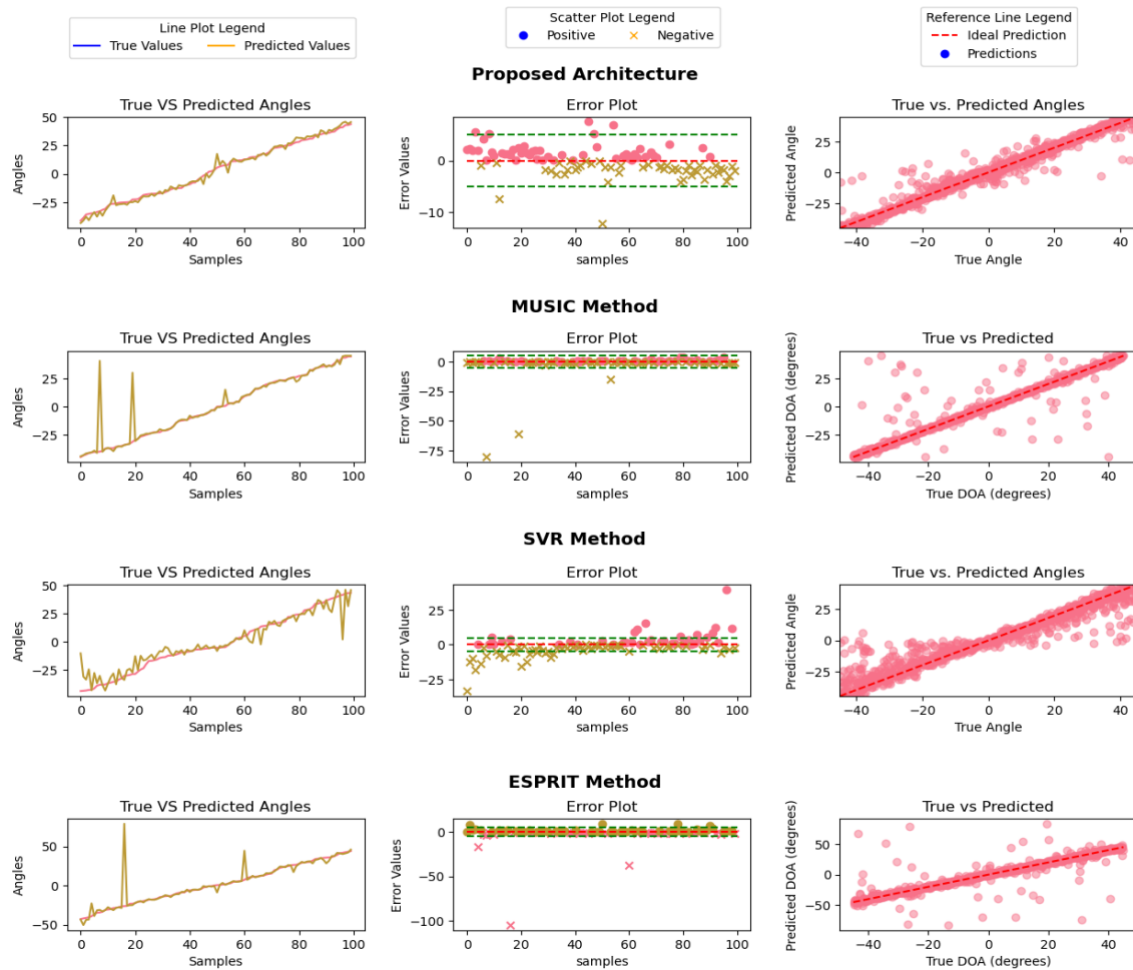


Figure 9: Performance Comparison based on (a) true vs predicted angles line plot, (b) errors, and (c) true vs predicted angles scatter plot.

The plots comparing the true DOA values with the predicted values for each method are given in Figure 9(a). It demonstrates that the predictions of the suggested method have a close relationship with the true values, whereas other methods have more prominent deviations. Figure 9(b) shows the distribution of the errors of all methods. The proposed approach has the least range of errors with most errors falling between -5 and 5, which means increased accuracy and reliability. Finally, Figure 9(c) indicates the scatter plot and reference line ($y = x$) to compare the actual and predicted DOA values. The suggested approach shows the most similar clustering along the reference line meaning that it performs better in the accurate estimation of DOA. The findings of the quantitative analysis and the visualizations are used to point out the strength and precision of the suggested methodology in relation to the conventional ones. This excellent performance leverages the combined strengths of LSTM and CNN, attention mechanisms, and advanced feature selection techniques. On the whole, the comparison highlights the ability of the proposed approach to obtain accurate and reliable estimation of the DOA.

5. Conclusions

This paper presents a new approach for DOA estimation based on an integration of LSTM Network, CNN and ANN with a multi-head attention mechanism based on positional encoding, and a MOGWO algorithm together with mRMR feature selection. The work considers DOA estimation a regression problem and as a result, continuous angle values are obtained. An in-depth analysis of the proposed technique showed considerable enhancement in the DOA estimation accuracy with respect to conventional algorithms like Support Vector Regression (SVR), MUSIC as well as ESPRIT. In particular, the proposed approach was able to attain an MAE of 2.15, MSE of 15.67, RMSE of 3.95 as well as an R^2 Score of 0.97, which highlights its superiority in complex contexts with a high level of noise and interference. Our framework has a number of important elements that can be credited with the increased accuracy. The model can be effective in the extraction of temporal and spatial features of the input data, by using both LSTM and CNN. Multi-head Attention with positional encoding mechanism enables the model to emphasize on the most important features whilst maintaining the positional information in the data. MOGWO integration with mRMR will make sure that the most informative and non-redundant features are chosen.

Although these advancements are made, there are shortcomings to our approach. The framework's baseline evaluation relies primarily on synthetic array scenarios. Real-world acoustic or electromagnetic signal fields exhibit localized uncalibrated manifold phase errors and multi-path reflections. Future extensions will focus on hardware prototyping and deployment on physical array signal processing hardware to validate generalized model resilience. Although these advancements are made, there are shortcomings to our approach. The sample size (5000) was used to carry out the experiments, and it can be improved by increasing the size of input. In addition, the work has not been proven to work on real-world data and therefore performance can be different. Finally, the optimization of the Artificial Neural Network (ANN) hyperparameters was not carried out which might possibly result in further more accurate DOA estimates. Further research will focus on overcoming these shortcomings by seeking a bigger sample to train and validate the suggested approach. The other direction is to test the proposed method with real-world data to confirm its strength and generalization to other settings and signal conditions. The idea of using hyperparameter tuning techniques to tune the ANN is another avenue that can be pursued in the future.

Funding Statement: The author received no specific funding for this study.

Data Availability: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest regarding this study.

Authors contributions. Conceptualization: ZHM, AY; data curation and methodology: ZHM, SKH, AY; validation and visualization: ZHM, SMH; writing original draft preparation: SKH, AY; writing, review and editing: ZHM, AY; supervision and project administration: SKH, AY. The authors had approved the final version.

AI use declaration

During the preparation of this work, generative AI tools were used solely for language editing purposes. The author reviewed and edited all content and take full responsibility for the final manuscript.

References

- [1] Kulkarni, S., Thakur, A., Soni, S., et al., (2025). "A comprehensive review of direction of arrival (DoA) estimation techniques and algorithms". *Journal of Electronics and Electrical Engineering*, 4(1), 138-186.
- [2] Ge, S., Li, K., and Rum, S. N. B. M. (2021). "Deep learning approach in DOA estimation: a systematic literature review". *Mobile Information Systems*, 2021, 1-14.
- [3] Sagar, M. S. I., Ouassal, H., Omi, A. I. et al., (2021). "Application of machine learning in electromagnetics: mini-review". *Electronics*, 10(22), 2752.
- [4] Ruan, N., Wang, H., Wen, F. et al., (2022). "DOA estimation in B5G/6G: trends and challenges". *Sensors*, 22(14), 5125.
- [5] Cui, M., and Dai (2022). "Channel estimation for extremely large-scale MIMO: far-field or near-field? IEEE Transactions on Communications, 70(4), 2663-2677".
- [6] Guan, H., Ding, S., Dai, W. et al., (2025). "Low complexity DOA estimation based on weighted noise component subtraction for smart-home application". *Applied Acoustics*, 231, 110490.
- [7] Oumar, O. A., Siyau, M. F., and Sattar, T. P., "Comparison between music and esprit direction of arrival estimation algorithms for wireless communication systems". In *The First International Conference on Future Generation Communication Technologies: IEEE*, 99-103. 2012.
- [8] Eranti, P. K., and Barkana, B. D. (2022). "An overview of direction-of-arrival estimation methods using adaptive directional time-frequency distributions". *Electronics*, 11(9), 1321.
- [9] Huang, M., Li, R., Zou, Y. et al., (2024). "A comprehensive review of metasurface-assisted direction-of-arrival estimation". *Nanophotonics*, 13(24), 4381-4396.
- [10] Elaraby, A., Saad, A., Karamti, H. et al., (2023). "An optimized deep learning approach for robust image quality classification". *Traitement Du Signal*, 40(4), 1573-1579.
- [11] Huang, H., Guo, S., Gui, G. et al., (2020). "Deep learning for physical-layer 5g wireless techniques: opportunities, challenges and solutions". *IEEE Wireless Communications*, 27(1), 214-222.
- [12] Chakrabarty, S., and Habets, E. A. P. (2019). "Multi-speaker DOA estimation using deep convolutional networks trained with noise signals". *IEEE Journal of Selected Topics in Signal Processing*, 13(1), 8-21.
- [13] Cooreman, P., Bohlender, A., and Madhu, N. (2023, September). "CRNN-based multi-DOA estimator: comparing classification and regression". In *Speech Communication; 15th ITG Conference*, 156-160, VDE.
- [14] Perotin, L., Défossez, A., Vincent, E., et al., (2019, October). "Regression versus classification for neural network-based audio source localization". In *2019 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, 343-347.

- [15] Xu, S., Wang, Z., Zhang, W. et al., (2024). "End-to-end regression neural network for coherent DOA estimation with dual-branch outputs". *IEEE Sensors Journal*, 24(3), 4047-4056.
- [16] Mosteghanemi, R., Benyarou, M., and Bendimerad, F. T., "A review of the state of the art of beamforming techniques and AI". In *2024 2nd International Conference on Electrical Engineering and Automatic Control (ICEEAC)*: IEEE, 1-6. 2024.
- [17] Wu, L., Liu, Z., and Huang, Z. (2019). "Deep convolution network for direction of arrival estimation with sparse prior". *IEEE Signal Processing Letters*, 26(11), 1688-1692.
- [18] Chen, Y., Wang, C., and Gao, Y. (2020). "Classification-based one-bit DOA estimation for sparse arrays". *IEEE Access*, 8, 204891-204901.
- [19] Kase, Y., Nishimura, T., Ohgane, T. et al., (2022). "Accuracy improvement in DOA estimation with deep learning". *IEICE Transactions on Communications*, E105.B(5), 588-599.
- [20] Hu, B., Liu, M., Yi, F. et al., (2020). "DOA robust estimation of echo signals based on deep learning networks with multiple type illuminators of opportunity". *IEEE Access*, 8, 14809-14819.
- [21] Zheng, S., Yang, Z., Shen, W. et al., (2024). "Deep learning-based DOA estimation". *IEEE Transactions on Cognitive Communications and Networking*, 10(3), 819-835.
- [22] Chen, P., Chen, Z., Liu, L. et al., (2024). "SDOA-Net: An efficient deep-learning-based DOA estimation network for imperfect array". *IEEE Transactions on Instrumentation and Measurement*, 73, 1-12.
- [23] Kassir, H. A., Kantartzis, N. V., Lazaridis, P. I. et al., (2024). "Improving DOA estimation via an optimal deep residual neural network classifier on uniform linear arrays". *IEEE Open Journal of Antennas and Propagation*, 5(2), 460-473.
- [24] Liu, A., Guo, J., Arnatovich, Y. et al., (2024). "Lightweight deep neural network with data redundancy removal and regression for doa estimation in sensor array". *Remote Sensing*, 16(8), 1423.
- [25] Xiong, Y., Liu, A., Gao, X. et al., "DOA estimation using deep neural network with regression". In *2022 5th international conference on information communication and signal processing (ICICSP)*: IEEE, 1-5. 2022.
- [26] Zhu, W. and Zhang, M., "A deep learning architecture for broadband DOA estimation". In *2019 IEEE 19th International Conference on Communication Technology (ICCT)*: IEEE, 244-247. 2019.
- [27] Chen, M., Gong, Y., and Mao, X. (2020). "Deep neural network for estimation of direction of arrival with antenna array". *IEEE Access*, 8, 140688-140698.
- [28] Guo, Y., Zhang, Z., Huang, Y. et al., (2020). "DOA estimation method based on cascaded neural network for two closely spaced sources". *IEEE Signal Processing Letters*, 27, 570-574.
- [29] Kassir, H. A., Zaharis, Z. D., Lazaridis, P. I. et al., (2022). "A review of the state of the art and future challenges of deep learning-based beamforming". *IEEE Access*, 10, 80869-80882.
- [30] Radoš, K., Brkić, M., and Begušić, D. (2024). "Recent advances on jamming and spoofing detection in GNSS". *Sensors*, 24(13), 4210.
- [31] Desai, D., and Mehendale, N. (2022). "A review on sound source localization systems". *Archives of Computational Methods in Engineering*, 29(7), 4631-4642.
- [32] Li, Q., Zhang, X., and Li, H., "Online direction of arrival estimation based on deep learning". In *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*: IEEE, 2616-2620. 2018.
- [33] Xie, Y., and Wang, B. (2023). "Direction-of-arrival estimation method based on neural network with temporal structure for underwater acoustic vector sensor array". *Sensors*, 23(10), 4919.
- [34] Tian, Q., Cai, R., Luo, Y. et al., (2025). "DOA estimation: LSTM and CNN learning algorithms". *Circuits, Systems, and Signal Processing*, 44(1), 652-669.
- [35] Li, J., Wang, X., Tu, Z. et al., (2021). "On the diversity of multi-head attention". *Neurocomputing*, 454, 14-24.
- [36] Cordonnier, J. B., Loukas, A., and Jaggi, M. (2020). "Multi-head attention: collaborate instead of concatenate. arXiv preprint arXiv:2006.16362.

- [37] Zheng, J., Ramasinghe, S., and Lucey, S. (2021). "Rethinking positional encoding". arXiv preprint arXiv:2107.02561.
- [38] Dokeroglu, T., Deniz, A., and Kiziloğlu, H. E. (2022). "A comprehensive survey on recent metaheuristics for feature selection". *Neurocomputing*, 494, 269-296.
- [39] Agrawal, P., Abutarboush, H. F., Ganesh, T. et al., (2021). "Metaheuristic algorithms on feature selection: a survey of one decade of research (2009-2019)". *IEEE Access*, 9, 26766-26791.
- [40] DADA, E., JOSEPH, S., OYEWOLA, D. et al., (2022). "Application of grey wolf optimization algorithm: recent trends, issues, and possible horizons". *Gazi University Journal of Science*, 35(2), 485-504.
- [41] Swarnkar, B., Pratyasha, P., and Padhy, A. P. (2020). "Large dimensional data reduction by various feature selection techniques: a short review". *International Journal of Computer Applications*, 176(41), 16-24.
- [42] Zhang, Y., and Wang, Z. (2023). "Feature engineering and model optimization based classification method for network intrusion detection". *Applied Sciences*, 13(16), 9363.
- [43] Barthelme, A., and Utschick, W. (2021). "DoA estimation using neural network-based covariance matrix reconstruction". *IEEE Signal Processing Letters*, 28, 783-787.
- [44] Ozanich, E., Gerstoft, P., and Niu, H. (2020). "A feedforward neural network for direction-of-arrival estimation". *The Journal of the Acoustical Society of America*, 147(3), 2035-2048.
- [45] Dubey, S. R., Singh, S. K., and Chaudhuri, B. B. (2022). "Activation functions in deep learning: a comprehensive survey and benchmark". *Neurocomputing*, 503, 92-108.
- [46] Elaraby, A., Nechaevskiy, A., and Saad, A. (2025). "Advancing lymphoma diagnosis in histopathology image classification using multi deep learning models". *Iraqi Journal for Computer Science and Mathematics*, 6(2).
- [47] Abdzaid, A. Y., Khadum, D. N., Al-Barrak, S. S. et al., (2026). "White blood cells classification based on a new strategy to cluster channels RGB of blood smear image using Resnet-18 model". *International Journal of Theoretical and Applied Computational Intelligence*, 2026, 120-131.